Grade Level/Course: Algebra I / Algebra II
Lesson/Unit Plan Name: Key Features of Graphs
Rationale/Lesson Abstract: SWBAT determine key features (increasing/decreasing, *x*- and *y*-intercepts, maximum/minimum, domain, range, and line of symmetry) from graphs.
Timeframe: 2-3 days (approx. 60 minute periods each day) — May extend depending on presentations. EXTENSION: + 1 day (extension activity takes one day)
Common Core Standard(s): F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★

Instructional Resources/Materials:

Day 1:
• Document camera and projector (important so that students can see the same graph that is on their paper and you can highlight/point out the key features)

• Copies of warm up

• Copies of student notes for Day 1

• Copies of exit tickets

Day 2/3:
• Document camera and projector

• Copies of warm up

• Copies of "Rubric for Posters and Presentations"

• Copies of the "Poster Functions" enough for 2 functions for each group of 4

• Small poster paper (2 per group)

• Poster creating materials (markers, colored pencils, tape, glue sticks)

• Whiteboards (not necessary)

• Copies of exit ticket

(EXTENSION) Day 4:
• Copies of "Graph Telephone" for each student

** Note: all copy-ready/student-friendly pages are found at the end of this packet on pages 8–21**

DAY 1

Warm-up (5 minutes):
Give students a half sheet with the vocabulary brainstorm on it (*Note: all copy-ready notes are found at the end of this lesson*). List the following vocabulary words on the board and instruct students to choose 2 of the words and take 3 minutes to write down everything they know about the word, draw a picture, and try to come up with a definition.

• *x*-intercept	• maximum	• symmetry
• *y*-intercept	• minimum	• domain
		• range

Spend 2 minutes debriefing the warm-up by asking students to raise their hand and give you some information about each of these words. Write down as much relevant information as you can next to the word on the board.

Student Notes: Day 1

Vocabulary (10 minutes):
If this is review for your students, ask them to help give you the definitions by either raising their hands or using equity sticks.
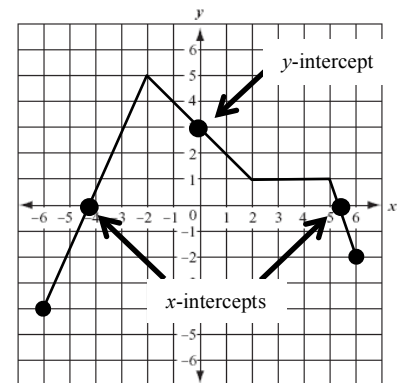
Domain: the input or *x*-values
Range: the output or *y*-values

Define the *x*- and *y*-intercepts for students and label those points on the graph to the right.

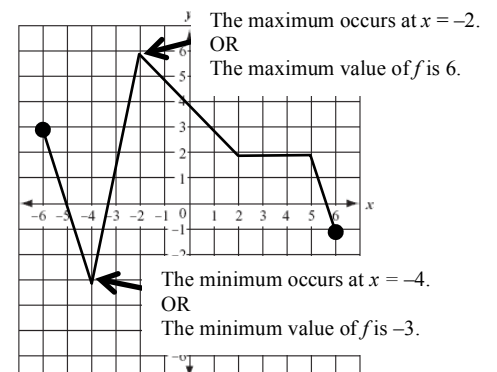*x*-intercept: where the graph of the function crosses the *x*-axis

y-intercept: where the graph of the function crosses the *y*-axis

Define maximum and minimum for students and label those on the graph.

The maximum of a function is the largest function value (output/range)

The minimum of a function is the smallest function value (output/range)

The maximum occurs at $x = -2$.
OR
The maximum value of *f* is 6.

The minimum occurs at $x = -4$.
OR
The minimum value of *f* is -3.

Define increasing and decreasing for students and highlight the parts
of the graph where each is happening.

Increasing: <u>going up (when read from left to right)</u>

Decreasing: <u>going down (when read from left to right)</u>
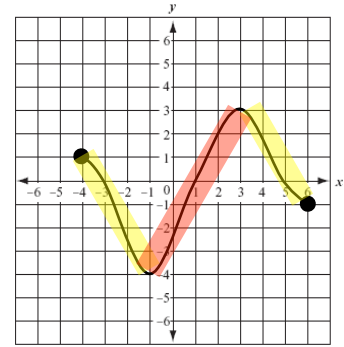
is where the graph is increasing.
is where the graph is decreasing.

Define axis of symmetry for students and guide them through labeling the axis of symmetry (if one exists)
for the figures below the definition.

<u>Axis of symmetry</u>: <u>a line through a shape so that each side is a mirror image</u>
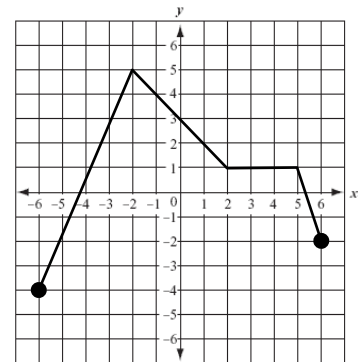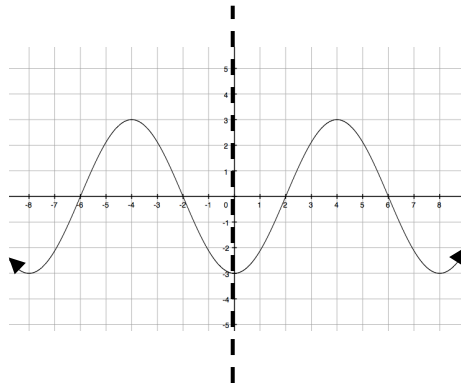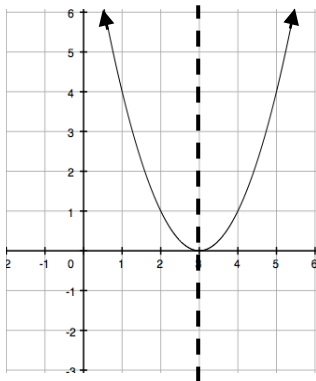
No axis of symmetry.

"I DO" (5 minutes):
1. Find the key features of the function $f(x)$, graphed here.
   a) Is the graph increasing or decreasing from $x = -2$ to $x = 0$?
      *Highlight the part of the graph that we are talking about, the section
      between $x = -2$ and $x = 0$. Say "If I look at the section of the graph
      in question, and I look from left to right, I see that the graph is sloping
      down. Therefore, on this interval, the graph is decreasing."*

   b) Is the graph increasing or decreasing from $2 < x < 3$?
      *Highlight the part of the graph that we are talking about, the section
      between $x = 2$ and $x = 3$. Say "If I look at this section of the graph from left to right, I see that the
      graph is sloping up. Therefore, on this interval, the graph is increasing."*

   c) x-intercept:
      *Remind students that the x-intercept is where the graph crosses the x-axis. Follow the x-axis with
      your finger until the graph crosses it. This happens in two places $x = -1$ and $x = 2$. Also, write these
      as the points (−1, 0) and (2, 0).*

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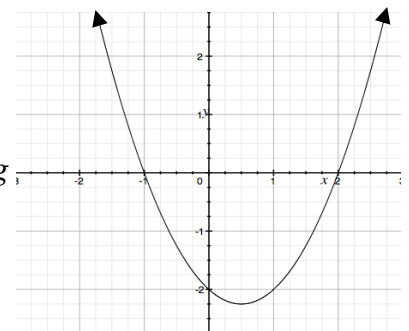
d) *y*-intercept:

   Remind students that the y-intercept is where the graph crosses the y-axis. Follow the y-axis with your finger until the graph crosses it. This happens once, at y = –2. Also, write as a point (0, –2).

e) Evaluate $f(1) =$

   Say "We are trying to find f of 1. This means where the graph is (or the y-value of the graph) when x = 1." Find x = 1 with your finger and follow it down to meet the graph. Say "This is the point (1, –2). When x = 1, the function is at –2. Therefore, f of 1 is negative 2."

f) Maximum:

   Remind students that a maximum is the largest value of the function, or the largest range value. Looking at this graph, it has arrows at the top, which means the graph extends to positive infinity. Therefore, this function does not have a maximum.

g) Minimum:

   Remind students that a minimum is the smallest value of the function, or the smallest range value. Looking at this graph, and tracing it with my finger, I can see that the lowest it will get is when y is a little smaller than –2. I can estimate this minimum value as –2.25 and say that f(x) has a minimum of –2.25 and that minimum occurs at $x \approx 0.5$.

h) Domain:

   The domain is all the possible x-values for this function. I start on the left and look towards the right. Because of the arrows on the left side of this graph, I know that it will extend towards the left until negative infinity. The graph is continuous through until the arrow on the right. This means that it will extend towards the right until positive infinity.
   The domain can be represented by $(-\infty, \infty)$ or $-\infty < x < \infty$.

i) Range:

   The range is all the possible y-values for this function. I start at the bottom and look upward. The graph does not begin until its minimum at $y \approx -2.25$. The graph is continuous through until the arrows on the top. This means that it will extend upward until positive infinity.
   The range can be represented by $[-2.25, \infty)$ or $-2.25 \leq x < \infty$.

j) Axis of symmetry?

   There is an axis of symmetry for this function. It passes through the graph at its minimum. It can be approximated by the line x = 0.5. (It is enough at this point for students simply to draw the axis of symmetry.)

"WE DO" (10 minutes):

For problem #2, guide students through finding all of the key features using equity sticks. Ask questions like:

"What are we looking for here?"

"How can we tell if a graph is increasing?"

"Name one interval where the graph is increasing. Is there another?"

"Name that interval in another way." Emphasize set notation $(-\infty, 7]$ vs. inequalities $-\infty < y \le 7$

"Where can we find the y-intercept?"

"Does this function have a y-intercept?"

"How do you know?"

"Where is it?"

"Can you write that as a point? Why or why not? What is the point?"…etc.

For problem #3, have students work on this in pairs or groups then use equity sticks to call on someone to share what they thought about each part (a – e) of the question. If there is confusion, ask students to come up to the document camera to point out the parts of the graph in question.

WE DO Solutions:

2. Find the key features of the function $g(x)$.

    a) **Where is the graph increasing?**

        The graph is never increasing.

    b) **y-intercept**: $y = -6$ or $(0, -6)$

    c) **x-intercept:** $x = -2$ or $(-2, 0)$

    d) **Find $g(3) = 1$**

    e) **Maximum:** Appears that $g(x)$ has a maximum value of 2.

    f) **Minimum:** There is no minimum on this graph.

    g) **Domain:** $(-\infty, \infty)$ or $-\infty < x < \infty$

    h) **Range:** $(-\infty, 2)$ or $-\infty < y < 2$

    i) **Axis of symmetry?** There is no axis of symmetry.

3. Find the key features of the function $k(x)$.

    a) **Is the graph increasing from $x = -4$ to $x = -1$?** No. The graph is decreasing from $x = -4$ to $x = -1$.

    b) **x-intercept:** $x = -4$ and 2 or $(-4, 0)$ and $(2, 0)$.

    c) **y-intercept:** $y = -2$ or $(0, -2)$

    d) **Find $k(-1) = -3$**

    e) **Maximum:** There is no maximum on this graph.

    f) **Minimum:** The minimum value of $k(x)$ is -3. Minimum occurs at $x = -1$.

    g) **Domain:** The domain is all real numbers or $(-\infty, \infty)$ or $-\infty < x < \infty$

    h) **Range:** $[-3, \infty)$ or $-3 \le y < \infty$

    i) **Axis of symmetry?** The axis of symmetry is at $x = -1$. (Ok if students say that it exists and draw the line on the graph.)

"YOU TRY" (20 minutes):
Give students 8 minutes to work on problems #4 – 6. Allow students to work in groups or partners (whatever your "You Try" style is).

At the end of the time, randomly select 6 different students to come to the document camera and present their solutions (break apart the questions so that 6 students tackle problems 4a-d, 4e-h, 5a-d, 5e-i, 6a-d, 6e-i).

YOU TRY Solutions:

4.

a) The graph is decreasing everywhere.
$(-\infty, \infty)$ or $-\infty < x < \infty$

b) $y = 4$ or $(0, 4)$

c) $d(4) = -4$

d) There is no maximum.

e) There is no minimum.

f) $(-\infty, \infty)$ or $-\infty < x < \infty$

g) $(-\infty, \infty)$ or $-\infty < y < \infty$

5.

a) $0 < x < \infty$

b) $y = 3$ or $(0, 3)$

c) $x \approx -2.5$ and 2.5 or the approximate points $(-2.5, 0)$ and $(2.5, 0)$

d) $j(-2) = 1$

e) Maximum value of 3. Occurs at $x = 0$.

f) There is no minimum.

g) $(-\infty, 3]$ or $-\infty < y \leq 3$

h) $(-\infty, \infty)$ or $-\infty < x < \infty$

i) Axis of symmetry at $x = 0$.

6.

a) $(-\infty, -2)$ and $(-1, \infty)$ OR $-\infty < x < -2$ and $-1 < x < \infty$

b) There is no x-intercept

c) $y = -3$ or $(0, -3)$

d) There is no maximum.

e) There is no minimum.

f) $h(-1) = 2$

g) $(-\infty, \infty)$ or $-\infty < x < \infty$

h) $(-\infty, \infty)$ or $-\infty < y < \infty$

i) There is no axis of symmetry

Exit ticket (10 minutes):
Decide which exit ticket you would like to administer (or administer both). Have students complete the exit ticket in 3 minutes. Then, switch exit tickets for peer grading. At the end of peer grading, switch exit tickets back and have students look at their scores. Ask students to raise their hands for each part they got correct (call out "who got part 'a' correct?") and record for re-teach/revisit use tomorrow.

Exit ticket Solutions:
a) $y = -4$ or $(0, -4)$
b) $x = -2$ and 2 or $(-2, 0)$ and $(2, 0)$
c) Increasing
d) Minimum value of -4. Minimum occurs at $x = 0$.
e) $g(-3) = 5$

Warm-up (10 minutes):
Give students warm-up half sheet (similar to the problems from yesterday) and give them 5 minutes to find all the requested key features. Then, have students come up to the document camera to illustrate those key features.

Group work (25 minutes):
Move students into groups of 4.
Give each group the following:
- 2 of the "Poster Functions"
- 4 copies of the "Rubric for Posters and Presentations"
- 2 pieces of poster paper
- poster making materials

Review with students all the material that needs to be on their poster. Answer any questions. Give groups 20 minutes to create two posters, one per function.

Tell the groups with function $d(x)$ that they do NOT have to discuss the axis of symmetry. The axis for symmetry for a linear function is all lines that are perpendicular to that function. If you choose to explain this, you can, but it is a little complex for Algebra 1 students and can be skipped.

Presentations (15 minutes):
Randomly select groups of students to come up and choose to present ONE of their two poster functions. Score each group based on the Rubric for Posters and Presentations. Have other students score the posters and presentations, based on the rubric they have in front of them. For immediate feedback, ask students to show on their whiteboards the score they gave the presenting group.

Exit ticket (10 minutes):
Have students complete the exit ticket in 4 minutes. Then, switch exit tickets for peer grading. At the end of peer grading, switch exit tickets back and have students look at their scores. Ask students to raise their hands for each part they got correct (call out "who got part 'a' correct?") and record for re-teach/revisit use tomorrow.

Exit ticket Solutions:
- **A)** False
- **B)** True
- **C)** False
- **D)** False
- **E)** True

EXTENSION
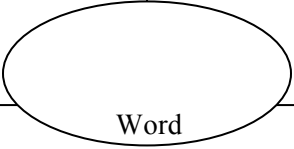
As an extension activity, you can play "Graph Telephone" with your students. Start by making 7 copies of the "Graph Telephone" graphic organizer (found on page 23 of this lesson). Draw 7 different functions on each of the "Teacher Start" coordinates. Make enough copies of these 7 pages for 1 page per student and distribute them so that the first student has the first function, second student has the second function, and so on. This will establish a "passing path" for students to follow when passing their graphs.
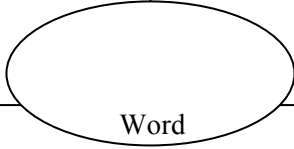
Have each student cut/tear their page in half so they have two strips of paper. Put the one with the "Teacher Start" on it in the front. Explain that each student should put his name on the strip as "Student 1". They then need to analyze the graph and describe all the key features as prompted in the "Student 1" box. After a certain time period (1-2 minutes), have Student 1 fold down the graph so that Student 2 will only be able to see the Student 1 box. Then have Student 1 pass (along the passing path you have established) to Student 2. Student 2 will sketch a graph based on the key features that Student 1 has described and, after 1-2 minutes, fold down Student 1's work, and pass to Student 3. And so on until Student 6 has drawn a final graph based on Student 5's description. **Take note that the second strip (strip on the right) is a continuation of the first strip (the strip on the left).**
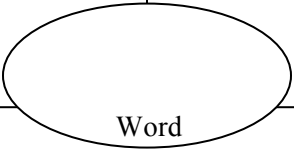
Student 6 should pass both strips back to Student 1. Give the class 3 minutes to answer the compare/contrast and error analysis questions on the final part of the strips. Lead a class discussion on how/if the graphs changed from Student 1 to Student 6. Discuss errors that occurred during this process.
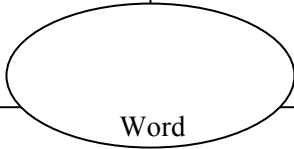
You can repeat this process as many times as you would like, with different functions that you can sketch on the "Graph Telephone" template.

Write down all that you know about this word.

Draw a picture of this word.

Word

Can you come up with a definition for this word?

Write down all that you know about this word.

Draw a picture of this word.

Word

Can you come up with a definition for this word?

Write down all that you know about this word.

Draw a picture of this word.

Word

Can you come up with a definition for this word?

Write down all that you know about this word.

Draw a picture of this word.

Word

Can you come up with a definition for this word?

SWBAT recognize key features from a graph.

Name: _____

Date:_____ Period: _____

Domain: _____

Range: _____

_x_-intercept: _____

_y_-intercept: _____
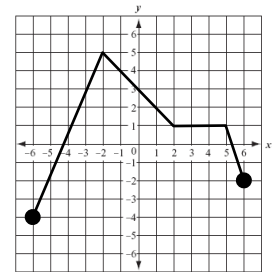
The <u>maximum</u> of a function is_____

The <u>minimum</u> of a function is_____
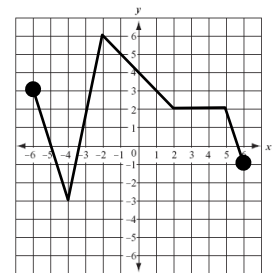
Increasing: _____
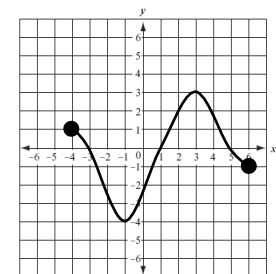
Decreasing: _____

Axis of symmetry: _____

If these figures have an axis of symmetry, draw it on.

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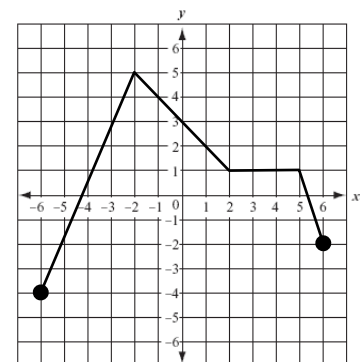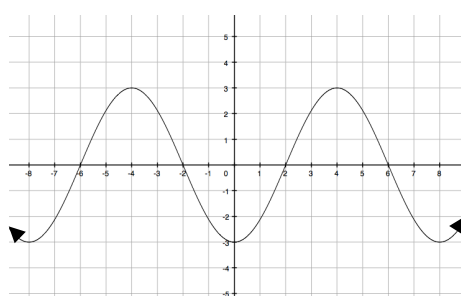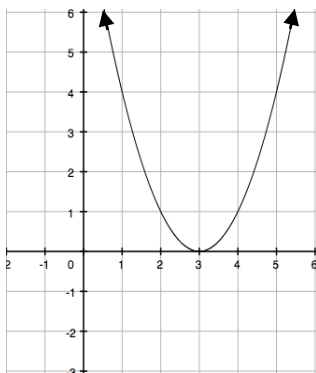
I DO

1. Find the key features of the function $f(x)$, graphed here.

   a) Is the graph increasing or decreasing from $x = -2$ to $x = 0$?

   b) Is the graph increasing or decreasing from $2 < x < 3$?

   c) x-intercept:

   d) y-intercept:

   e) Evaluate $f(1) =$

   f) Maximum:

   g) Minimum:

   h) Domain:

   i) Range:

   j) Axis of symmetry?
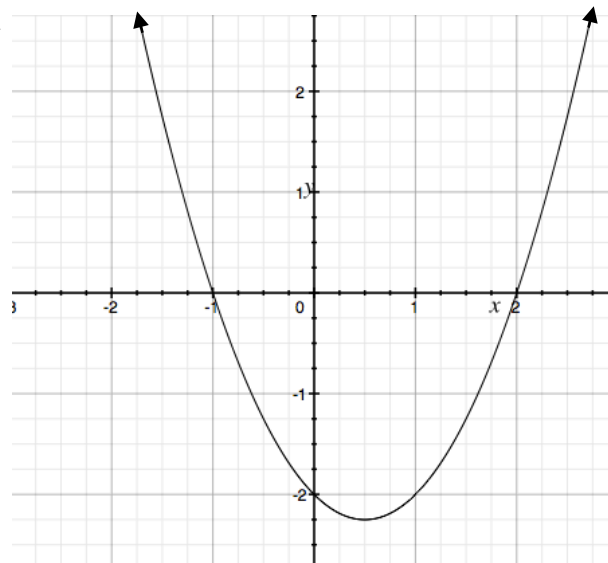
WE DO

2. Find the key features of the function $g(x)$ to the right.

   a) Where is the graph increasing?

   b) y-intercept:

   c) x-intercept:

   d) Find $g(3) =$

   e) Maximum:

   f) Minimum:

   g) Domain:

   h) Range:

   i) Axis of symmetry?

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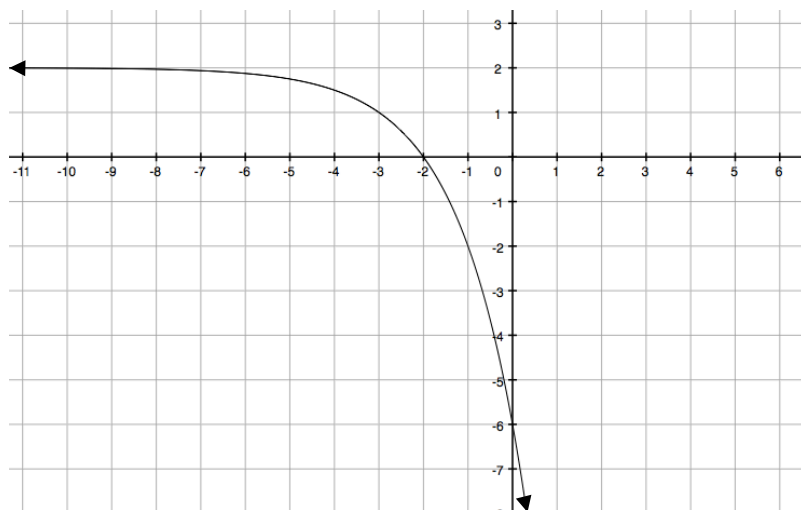
WE DO

3. Find the key features of the function $k(x)$ on the right.

a) Is the graph increasing from $x = -4$ to $x = -1$?

b) x-intercept:

c) y-intercept:

d) Find $k(-1) =$

e) Maximum:

f) Minimum:

g) Domain:

h) Range:

i) Axis of symmetry?
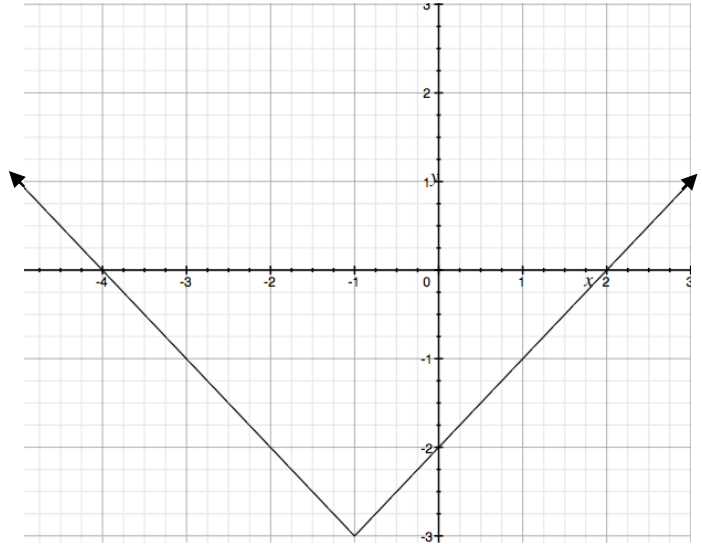
YOU TRY:

4. Find the key features of the function $d(x)$, graphed here.

a) Where is the graph decreasing?

b) y-intercept:

c) Find $d(4) =$

d) Maximum:

e) Minimum:

f) Domain:

g) Range:
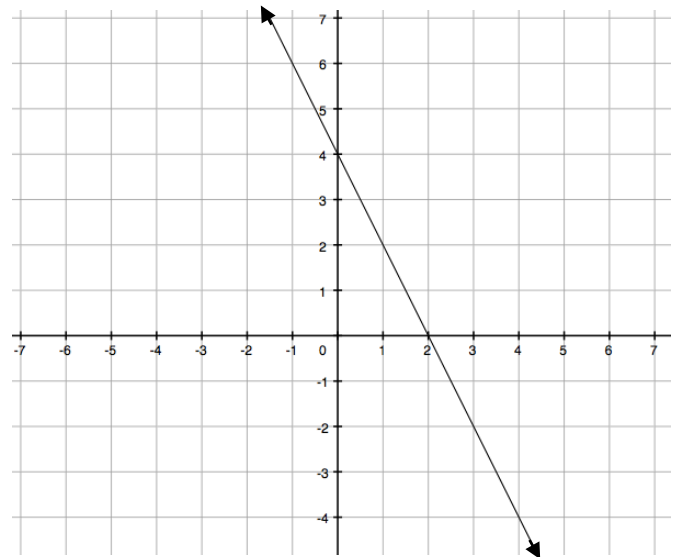
5. Find the key features of the function $j(x)$ to the right.

a) Where is the graph decreasing?

b) y-intercept:

c) x-intercept:

d) Find $j(-2) =$

e) Maximum:

f) Minimum:

g) Range:

h) Domain:

i) Axis of symmetry?
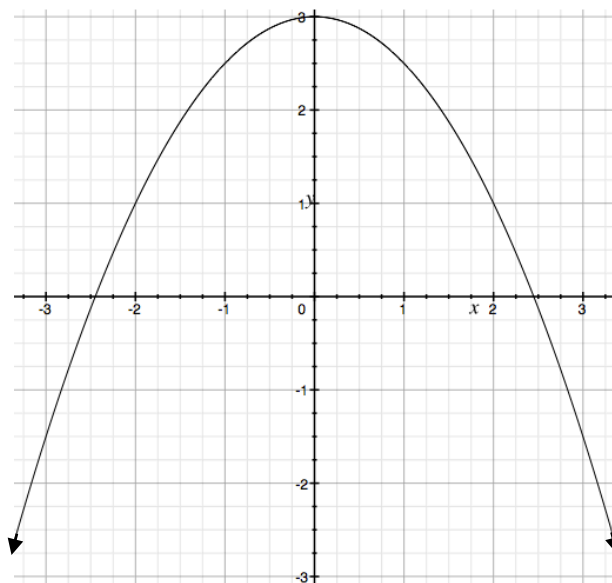
6. Find the key features of the function $h(x)$ here.

a) Where is the function decreasing?

b) x-intercept:

c) y-intercept:

d) Maximum:

e) Minimum:

f) Find $h(-1) =$

g) Domain:

h) Range:

i) Axis of symmetry?
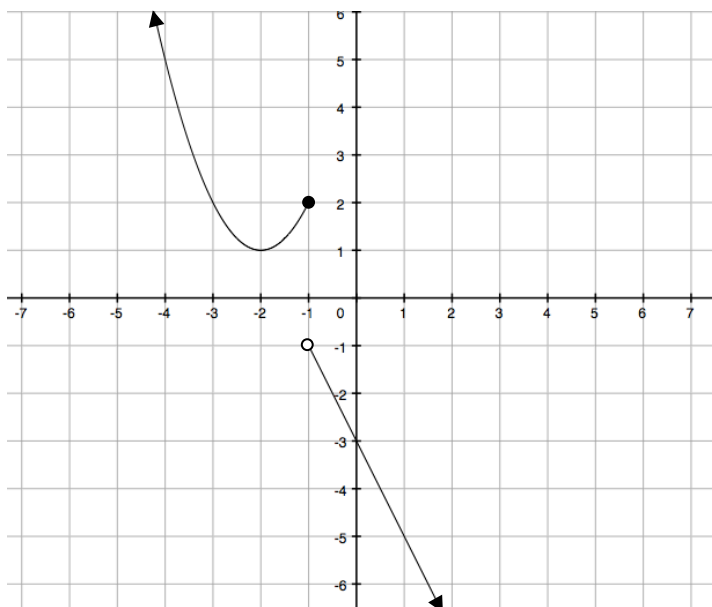
EXIT TICKET Name_____
  Date:_____ Period:____

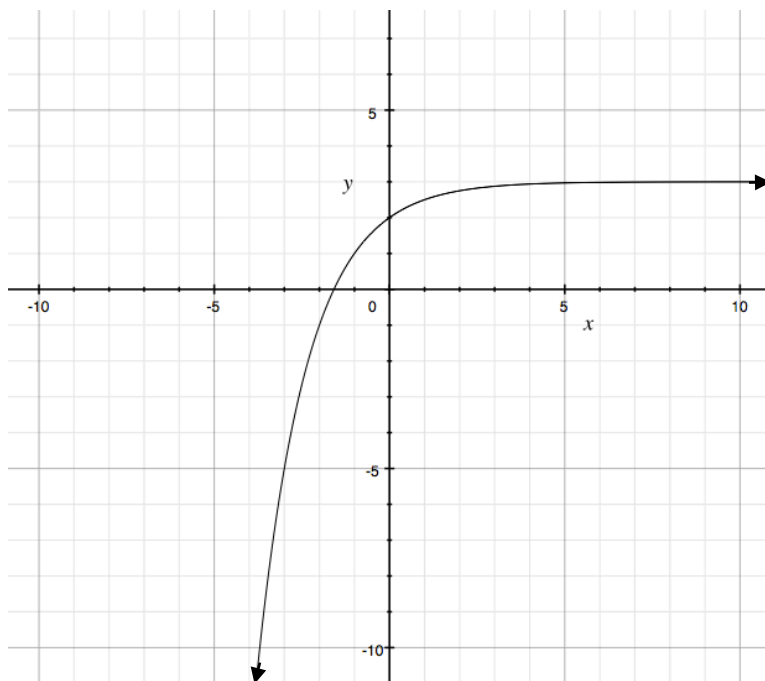Answer the following questions about the graph of the function $g(x)$ shown below.
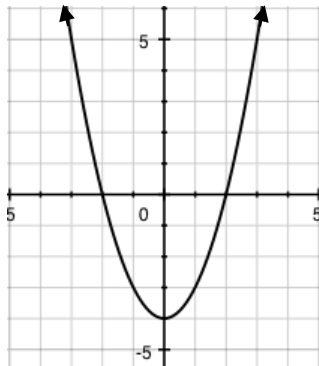
a) What are the y-intercept(s)?

b) What are the x-intercept(s)?

c) Is $g(x)$ increasing or decreasing on the interval $0 < x < 2$?

d) Does the graph have a minimum, maximum, both or neither? If so, where are these points?
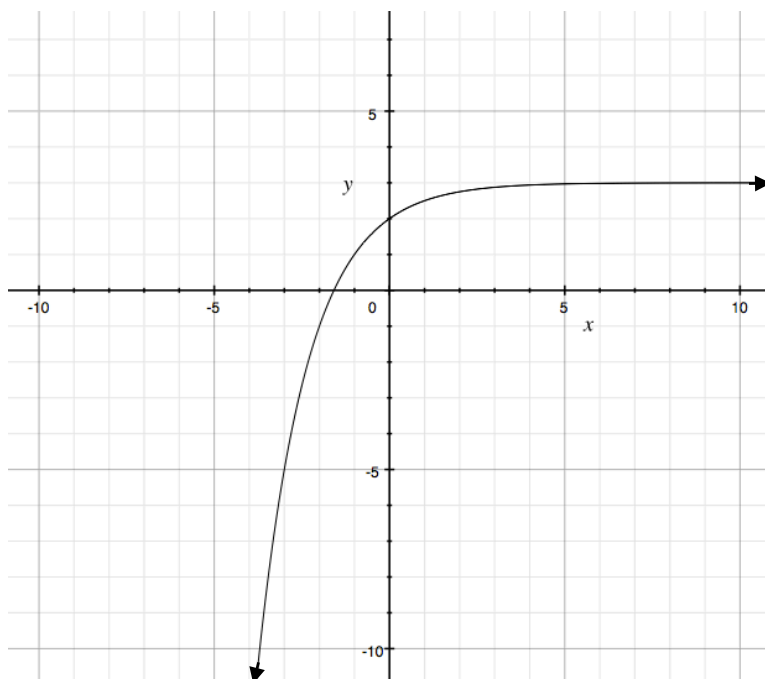
e) What is $g(-3)$?

EXIT TICKET Name_____
  Date:_____ Period:____

Answer the following questions about the graph of the function $g(x)$ shown below.

a) What are the y-intercept(s)?

b) What are the x-intercept(s)?

c) Is $g(x)$ increasing or decreasing on the interval $0 < x < 2$?

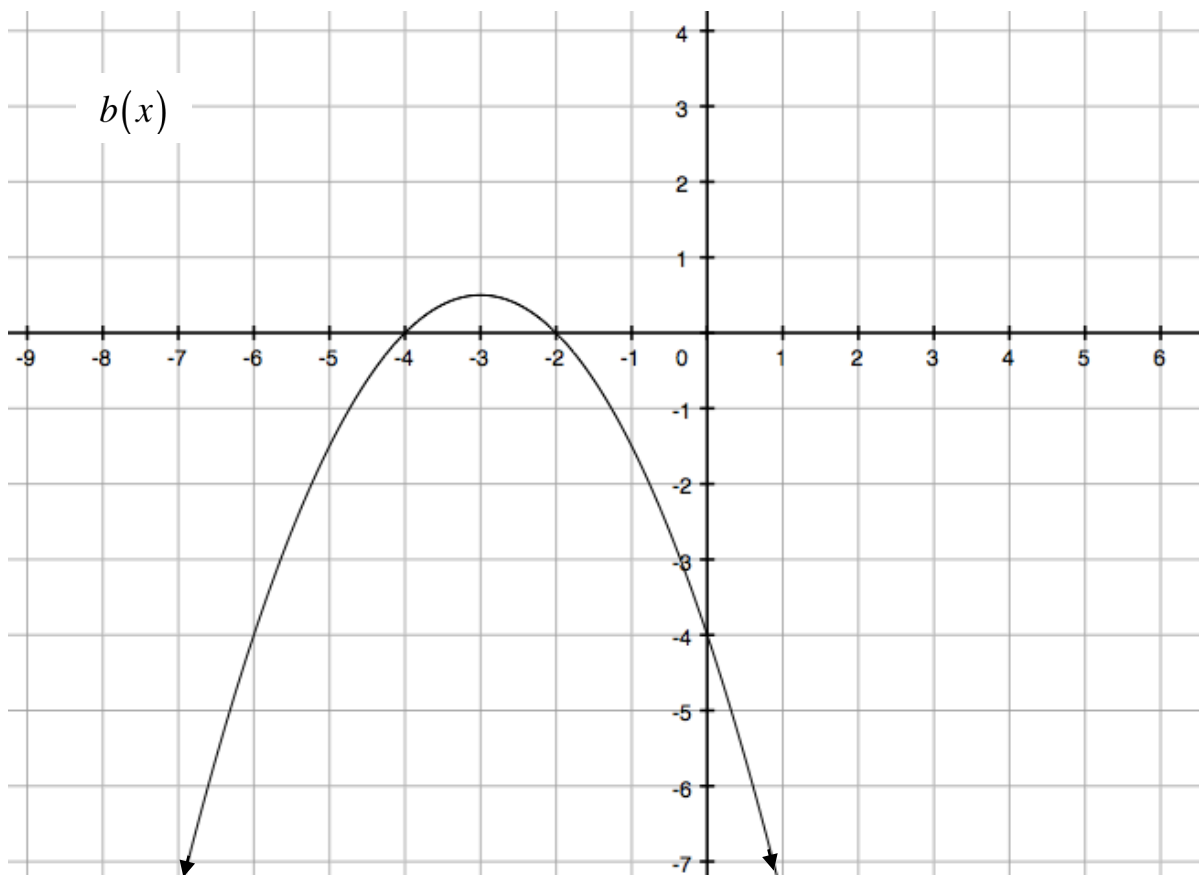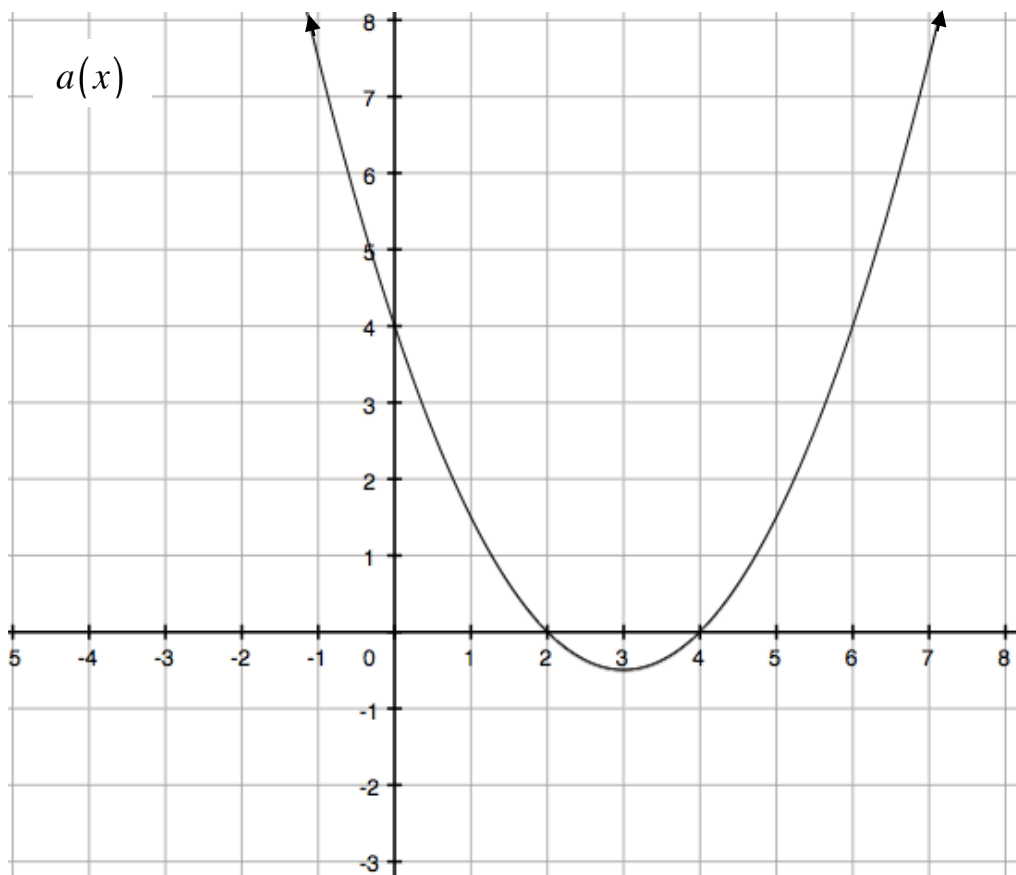d) Does the graph have a minimum, maximum, both or neither? If so, where are these points?
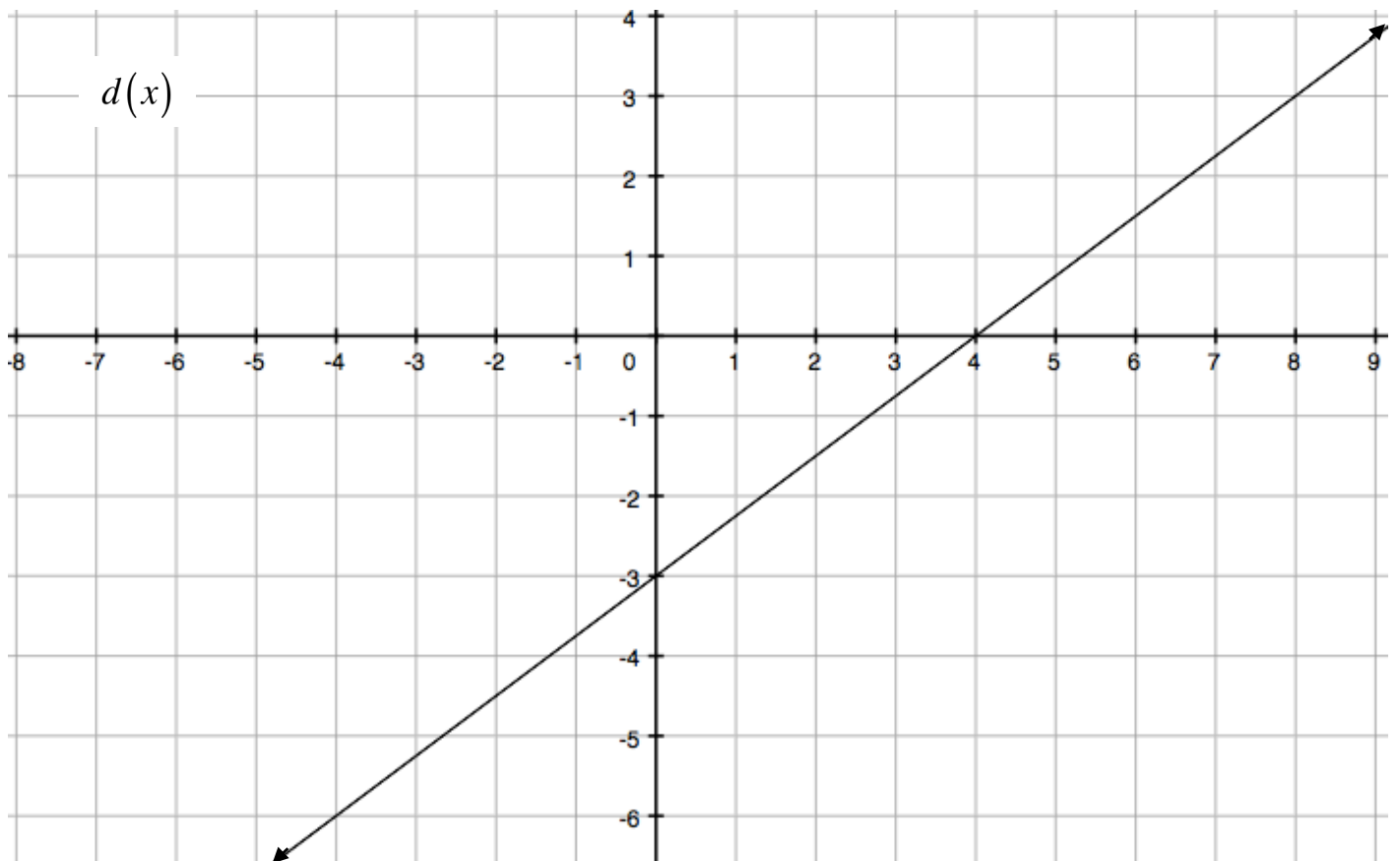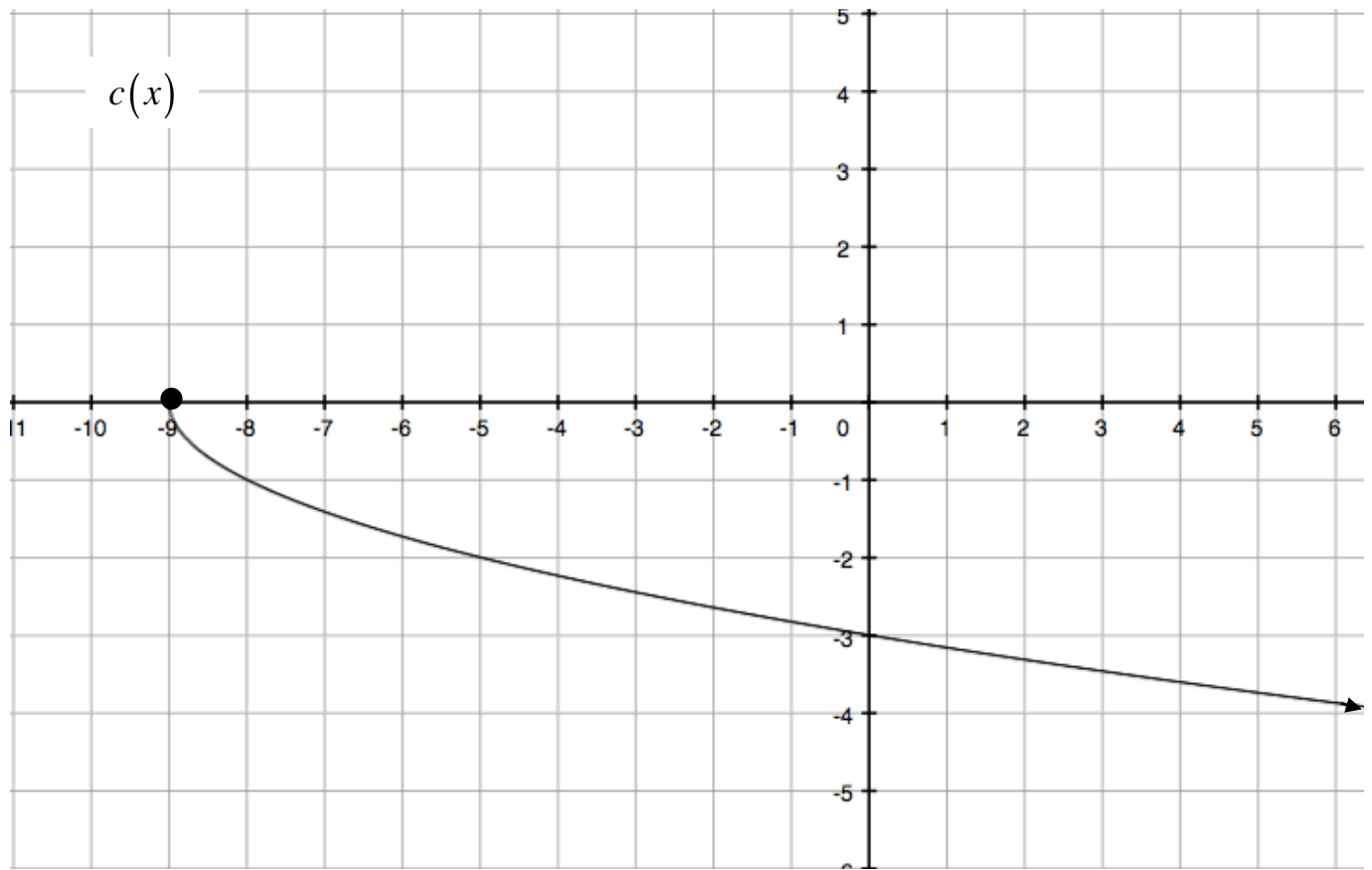
e) What is $g(-3)$?
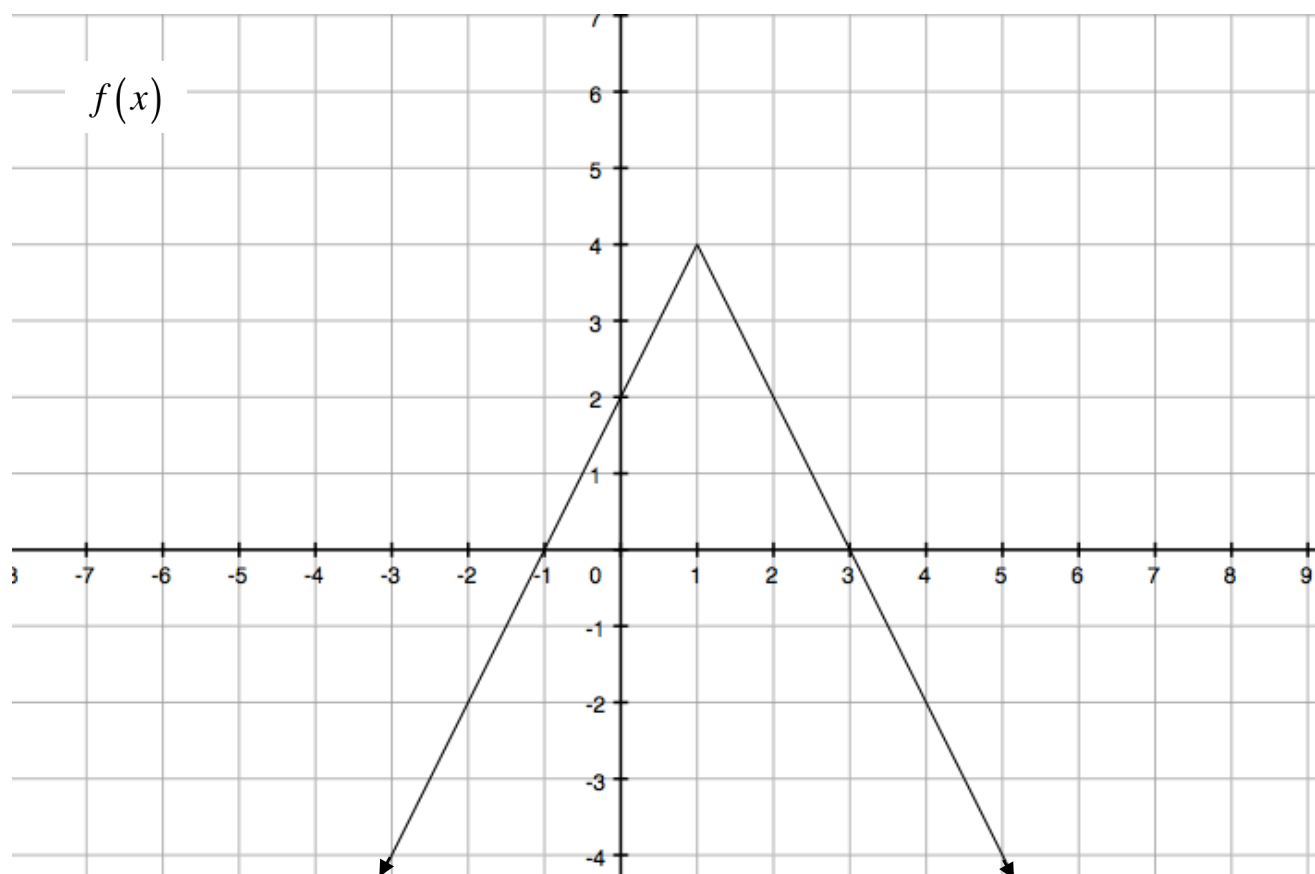
Answer the following questions about $f(x)$, graphed below.

a) Is the graph increasing or decreasing from $x = 0$ to $x = 4$?

b) y-intercept:

c) x-intercept:

d) $f(-1)=$

e) Range:

f) Domain:

g) Maximum:

h) Minimum:

Answer the following questions about $f(x)$, graphed below.
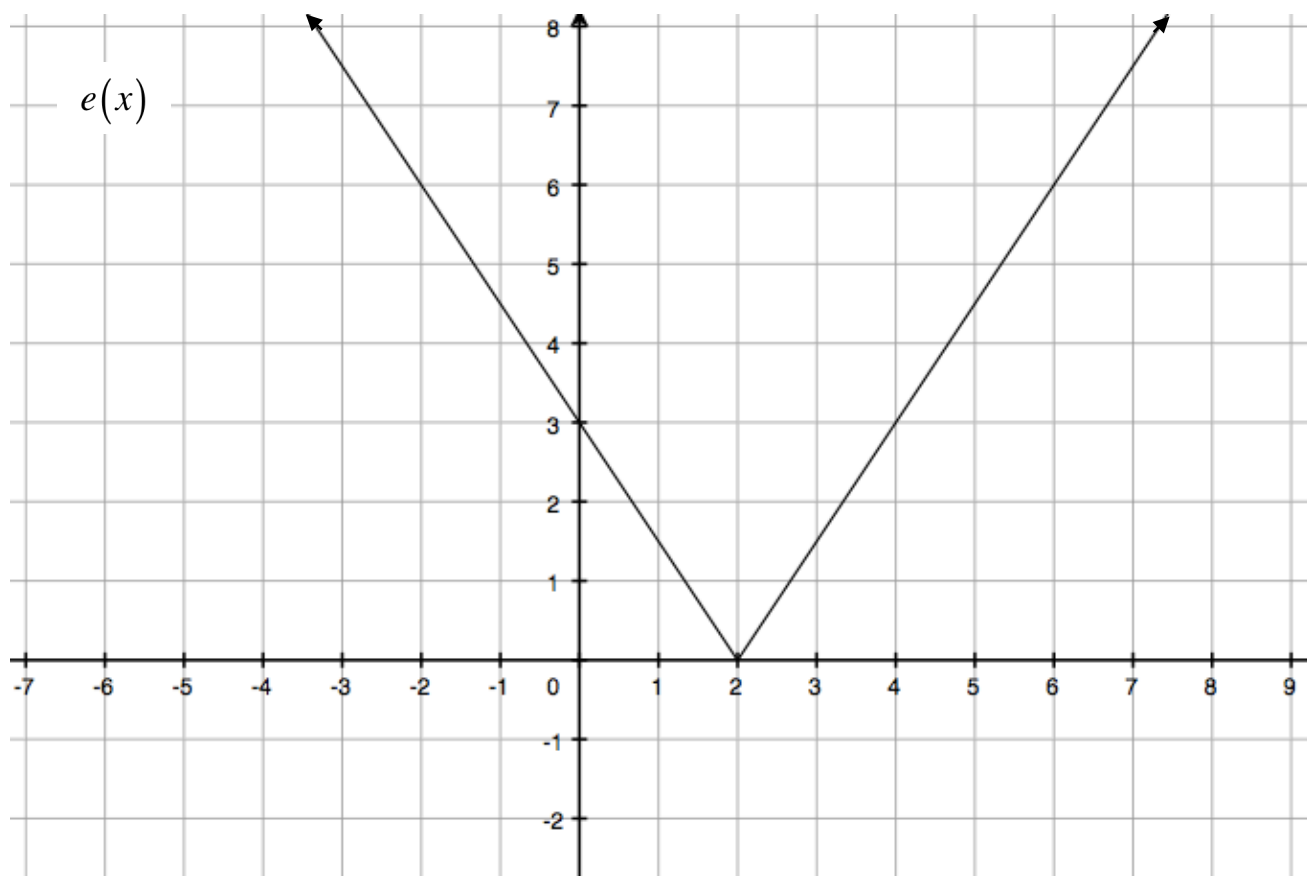
a) Is the graph increasing or decreasing from $x = 0$ to $x = 4$?

b) y-intercept:

c) x-intercept:

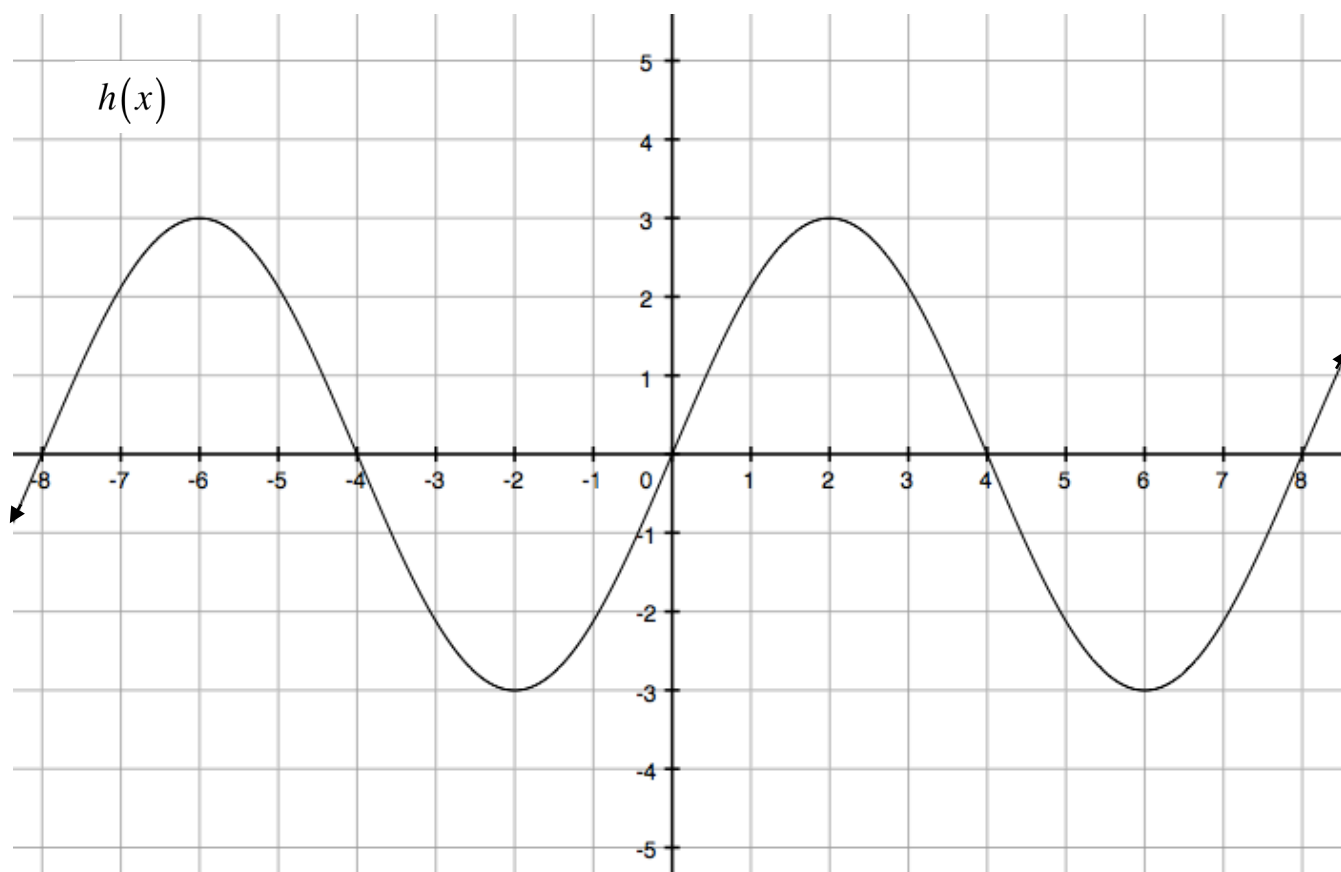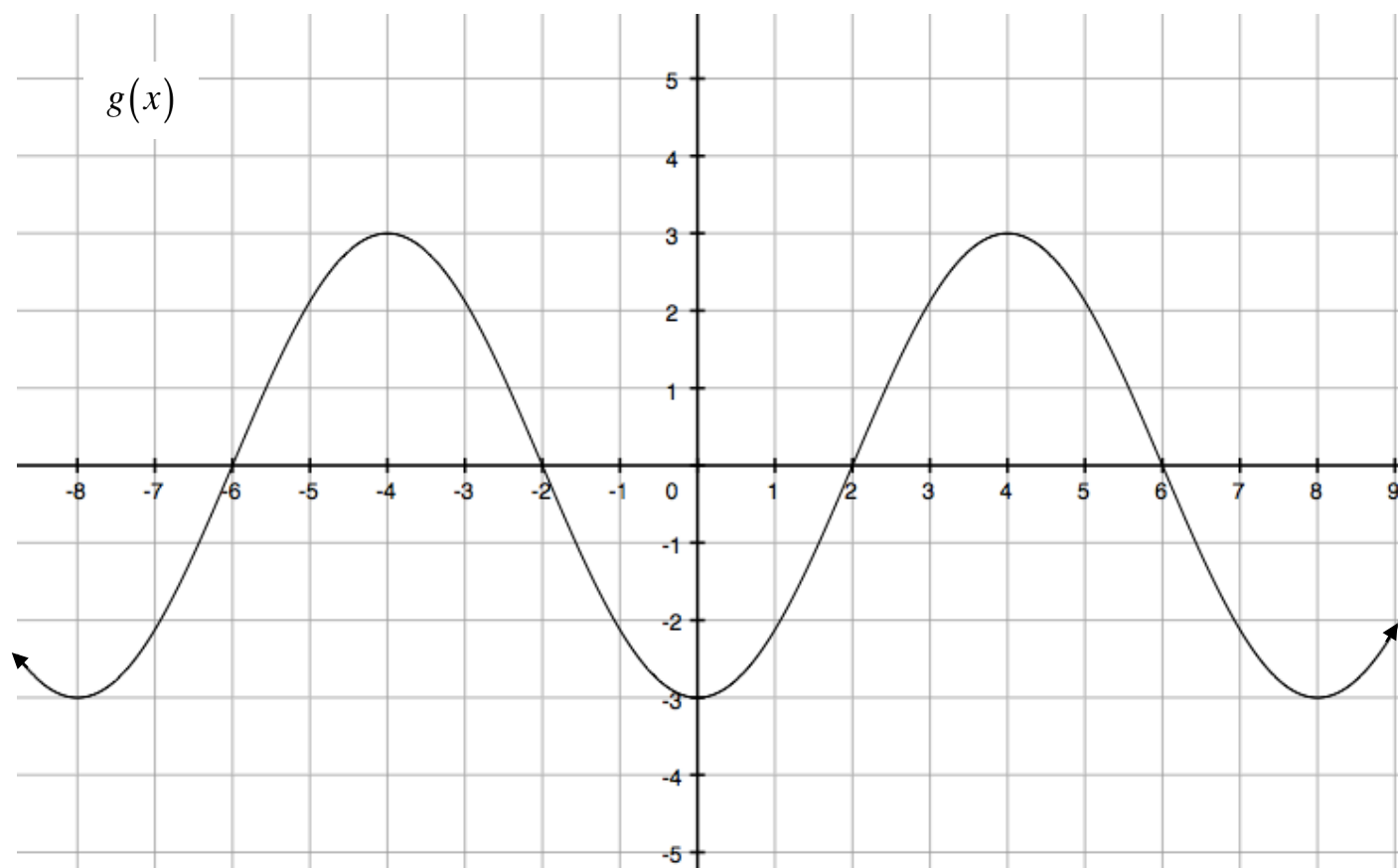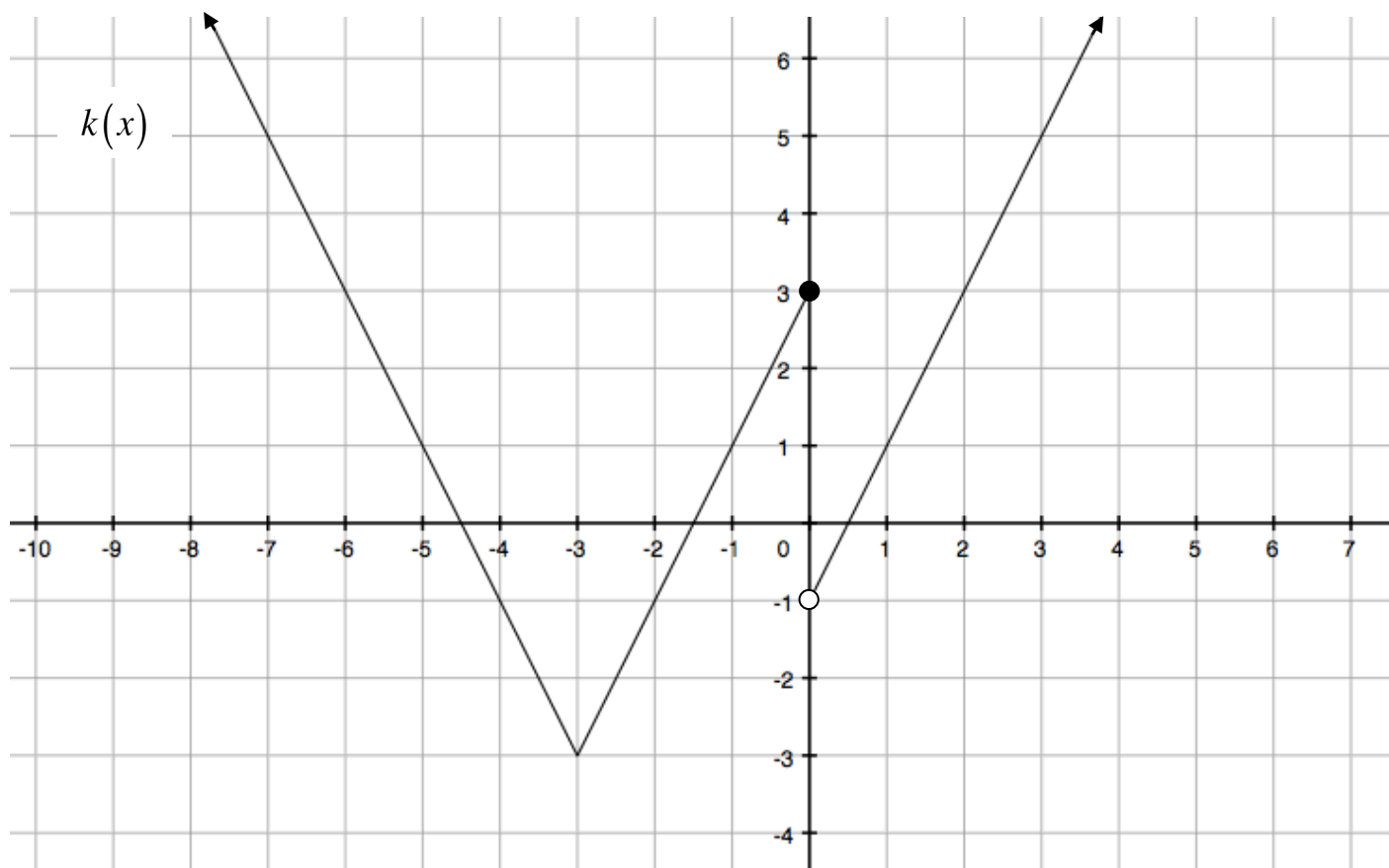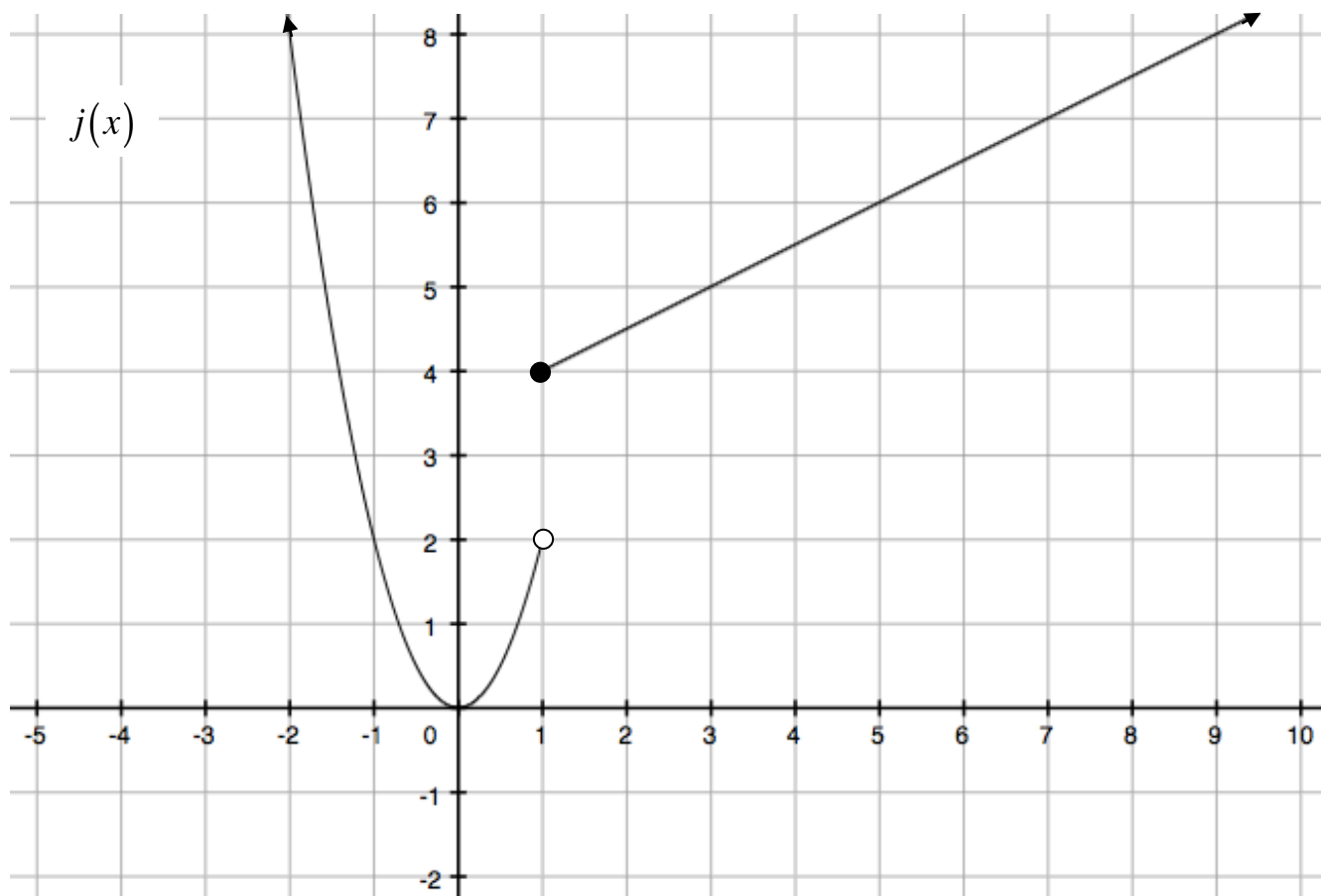d) $f(-1)=$

e) Range:

f) Domain:

g) Maximum:

h) Minimum:

$a(x)$

$b(x)$

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$c(x)$

$d(x)$

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$e(x)$

$f(x)$

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$g(x)$

$h(x)$

$j(x)$

$k(x)$

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Rubric for Posters and Presentations: Day 2

You must address ALL of the following 9 key features for your graph(s):

1) *x*-intercept
2) *y*-intercept
3) Intervals where the graph is increasing
4) Intervals where the graph is decreasing
5) Domain

6) Range
7) Maximum
8) Minimum
9) Axis of symmetry

	0	1	2	3	4	Score
Content	All key features are either missing or incorrect.	More than a few key features are missing OR most key features are incorrect.	A few key features are missing, but all identified are correct.	All 9 key features are identified but some are incorrect.	All 9 key features are identified and correct.	
Clarity of information	Students do not speak clearly AND do not address all of the content.	Students sometimes speak clearly OR students do not address all of the content.	Students sometimes speak clearly and address all of the content.	Students mostly speak clearly and address all parts of the content.	Students always speak clearly and address all parts of the content.	
Design/ Layout	No poster.	Design lacks creativity AND is not interesting.	Design either lacks creativity OR is not interesting.	Design is somewhat creative and relatively interesting, but could be improved.	Design is interesting and shows above average creativity.	
Group participation	No presentation.	1 or 2 members presented.	3 out of 4 members presented, but only minimally.	All members presented, but not equally.	Presentation is done equally by all 4 team members.	
						/16
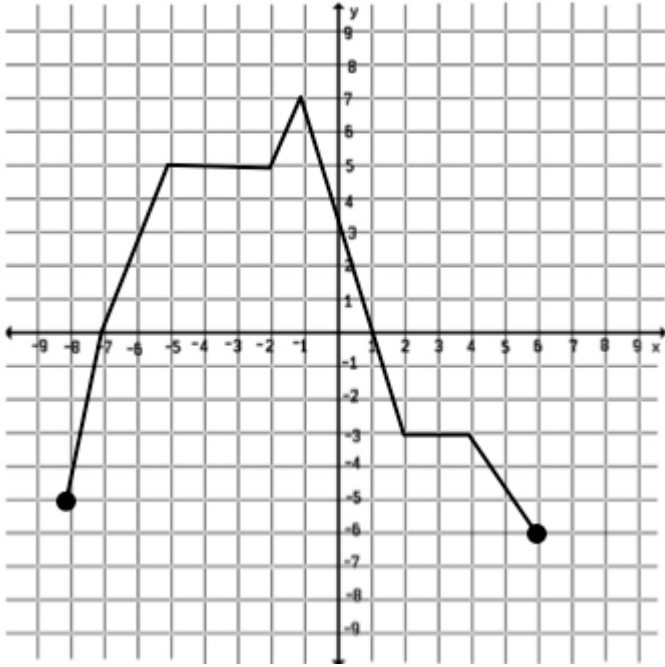
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EXIT TICKET   Name_____

Date:_____ Period:____

A function $p(x)$ is shown on the graph below.

Determine whether the following statements are True or False.

A) $p(x)$ is increasing on the interval $-1 < x < 0$

◯ True   ◯ False

B) $p(x)$ has an x-intercept at $x = 1$

◯ True   ◯ False

C) $p(x)$ has a y-intercept at $y = -7$

◯ True   ◯ False

D) $p(x)$ has a minimum at $x = -8$

◯ True   ◯ False

E) $p(2) = -3$

◯ True   ◯ False

EXIT TICKET   Name_____

Date:_____ Period:____
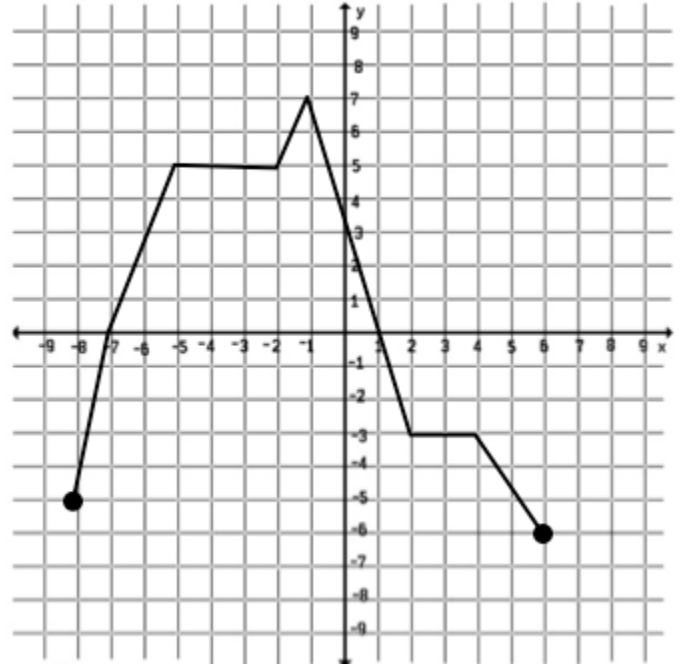
A function $p(x)$ is shown on the graph below.

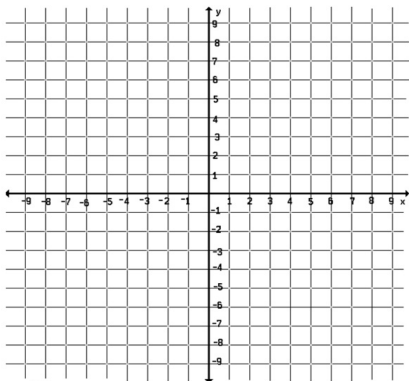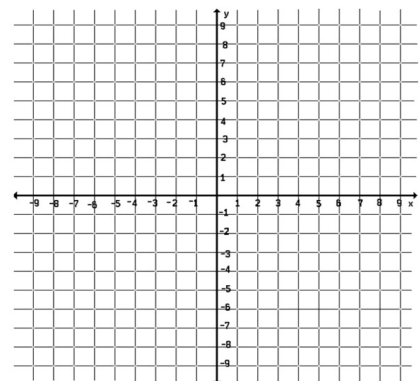Determine whether the following statements are True or False.

A) $p(x)$ is increasing on the interval $-1 < x < 0$

◯ True   ◯ False

B) $p(x)$ has an x-intercept at $x = 1$

◯ True   ◯ False

C) $p(x)$ has a y-intercept at $y = -7$

◯ True   ◯ False

D) $p(x)$ has a minimum at $x = -8$

◯ True   ◯ False

E) $p(2) = -3$

◯ True   ◯ False

Teacher Start:

Student 4: Name_____

Student 1: Name_____

a) Increasing where?   e) *x*-intercept?

                                    f) *y*-intercept?

b) Decreasing where?

                                    g) Domain:

c) Maximum?

                                    h) Range:

d) Minimum?

                                    i) Axis of symmetry?

Student 5: Name_____

a) Increasing where?   e) *x*-intercept?

                                    f) *y*-intercept?

b) Decreasing where?

                                    g) Domain:

c) Maximum?

                                    h) Range:

d) Minimum?

                                    i) Axis of symmetry?

Student 2: Name_____
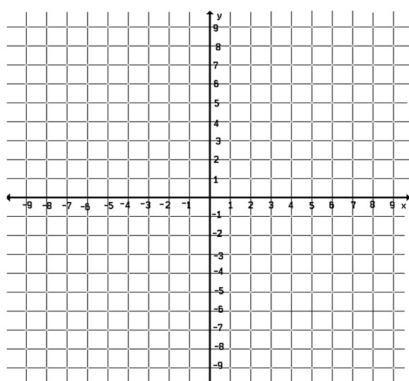
Student 6: Name_____
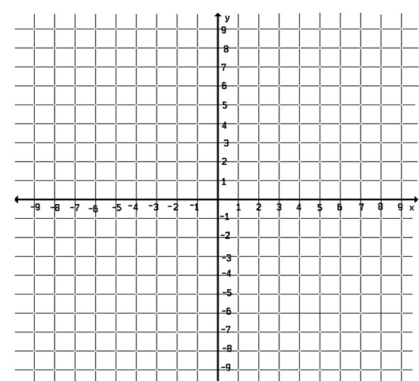
Student 3: Name_____

a) Increasing where?   e) *x*-intercept?

                                    f) *y*-intercept?

b) Decreasing where?

                                    g) Domain:

c) Maximum?

                                    h) Range:

d) Minimum?

                                    i) Axis of symmetry?

Give papers back to Student 1. Student 1 answers the following questions:

Does the last graph match the graph the teacher gave you?

What's different? What's the same?

Looking at everyone's work, were there mistakes? Where?